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Landau damping and free-electron laser interaction in storage rings

G. Dattoli[1], G. De Ninno[2,a], and M. Migliorati[3]

[1] ENEA, CP-65 00044 Frascati, Rome, Italy
[2] Sincrotrone Trieste, 34012 Trieste, Italy
[3] Università di Roma La Sapienza, 00161 Rome, Italy

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Abstract. In this paper we will discuss the conditions under which the free-electron laser interaction in a storage ring may reinforce the effect of the Landau damping, thus leading to the suppression of different types of instabilities. The problem will be discussed both by making use of general arguments and referring to the specific examples.

PACS. 41.60.Cr Free-electron lasers – 29.20.Dh Storage rings – 05.45.-a Nonlinear dynamics and nonlinear dynamical systems

1 Introduction

In a Free Electron Laser (FEL), a relativistic electron beam interacts with an electromagnetic field as it passes through a periodic magnetic structure forcing particles to move along sin-like trajectories and, consequently, to emit radiation. Depending on their starting phases, electrons go slower or faster after the interaction and this leads to a clustering further downstream. This microbunching is, in turn, the source of enhanced (coherent) radiation emission.

FEL oscillators provide intense, tunable, monochromatic and fully coherent radiation in the range from the infrared to the UV/VUV. In a storage-ring FEL (SRFEL) the light produced by the electron beam is stored in an optical cavity and amplified during the successive turns of the particles in the ring. The amplification is obtained to the detriment of the electron beam energy spread which becomes larger when the intracavity power grows. The heating of the electron bunch due to the laser onset leads to the reduction of the amplification gain until when it reaches the level of the cavity losses (laser saturation). Among oscillators, SRFELs present by far the more complex dynamics. Such complexity originates from the fact that, unlike a LINAC-based FEL, where the electron beam is renewed after each passage inside the interaction region, electrons are recirculated. As a result, at every light-beam energy exchange the system keeps memory of previous interactions. Moreover, the electron-beam dynamics and, as a consequence, the light evolution inside the optical cavity are generally influenced by beam interaction with the machine environment (e.g. the metallic wall of the ring vacuum chamber).

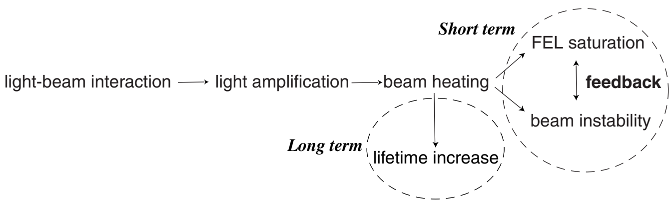

Fig. 1. Schematic diagram reproducing the dynamics of a SRFEL, including the mutual feedback with the instabilities generated by the electron-beam interaction with the machine environment.

The block diagram in Figure 1 reproduces the main features of the dynamics of a SRFEL, including its interplay with the instabilities generated by the electron-beam interaction with the machine environment. By taking the synchrotron damping time as reference, one can make a first, albeit crude, distinction between short and long-term effects. In fact, even though the physical mechanisms occurring at the two temporal scales can be traced back to a common root, such a distinction reveals to be helpful to develop an appropriate description of the various elements characterizing this quite entangled problem.

On the short-term scale, the electron-beam interaction with the electromagnetic field stored in the optical cavity leads to the light amplification and to the consequent increase of the electron-beam energy spread and bunch

[a] e-mail: giovanni.deninno@elettra.trieste.it

length. This heating effect is in turn responsible for the gain dilution and, therefore, for the FEL saturation. Generally speaking, the increase of the electron-beam dimensions is also the principal mechanism leading to an attenuation of instabilities associated with the particle density.

As to the long-term effect related to the SRFEL dynamics, one can observe [1] an increase of the Touschek lifetime, mainly due to the blow-up of the electron-beam transverse dimensions, which is in turn induced by the beam longitudinal heating through a nonzero dispersion function along the ring.

A number of papers [1–6] have been dedicated, in the recent past, to investigate the interplay between the laser dynamics and a particular class of electron-beam instabilities. This is, for example, the case of longitudinal instabilities of saw-tooth type for which the FEL onset and the instability manifestation have been shown to be competitive phenomena [6]. The instability itself is indeed responsible for a beam heating which, in turn, induces a concurrent decrease of the instability growth rate. When the heating generated by the FEL is dominant, the current threshold above which the instability develops is shifted and the instability has no chance to grow. Vice-versa, if the anomalous bunch lengthening generated by the instability is so large to reduce the FEL gain below the level of the optical cavity losses, the laser is no more able to develop and to counteract the instability. Using a similar argument, again based on the FEL-instability competition, it has been shown that the FEL may also counteract transverse electron-beam instabilities, such as the head-tail one [3].

The (longitudinal and transverse) beam heating due to the FEL onset leads to an increase of the amplitudes of the electron (synchrotron and betatron) oscillations. As a consequence, electrons experience stronger nonlinear components of the guiding fields and the beam synchrotron and betatron frequency spread is enhanced. This allowed to interpret the suppression of the saw-tooth instability due to the FEL onset as a genuine manifestation of stabilization through Landau damping [5,6]. As it is well-known [7–9], this process occurs when there are several oscillators whose natural frequencies are characterized by a given spread. In this case, an external force (as it can be the one induced by a coherent instability) in resonance with the system may provide energy to it without driving the oscillators to larger amplitudes. The result may be then a stable beam even in the presence of an active instability. The strength of the damping mechanism depends on the frequency distribution of the particles.

Aim of this paper is to show that the beneficial effect of the FEL in terms of strengthening of the Landau damping is not only limited to the case in which the laser onset and the instability are similar and, as a consequence, competitive phenomena. In fact, by means of general arguments, it will be shown that this holds for the whole class of coherent instabilities, both in the longitudinal and in the transverse plane, which may be damped through a suitable dilution of the beam dimensions. The analysis will be supported by a numerical study.

2 FEL-Landau damping interplay through a nonlinear field

In the transverse electron-beam phase space, the frequencies (tunes) spread depends on the amplitude of the betatron oscillation, due to the nonlinear components of the guiding field. In [10] this phenomenon has been studied for the case of octupolar field lenses. According to the results reported in this paper, the Landau damping manifests itself through an increase of the current threshold (i.e. the maximum particles' number), N_{th}, according to the relation

$$\Im(\bar{Z})N_{th} \propto \mp \frac{1}{[G(u)^2+1]} \frac{e^u}{u} B\sigma_x^2, \tag{1}$$

with

$$G(u) = \mp \left[\int_{-\infty}^{\infty} \frac{e^{-\xi}}{\xi} \mathrm{d}\xi + \frac{e^u}{u} \right] \tag{2}$$

and

$$u = \frac{\nu - \nu_0}{2B\sigma_x^2}. \tag{3}$$

Here $\Im(\bar{Z})$ is the imaginary part of the effective impedance and B is linked to the strength of the nonlinear field; u accounts for the coherent tune shift induced by the nonlinearity, ν and ν_0 being the coherent and the zero-amplitude tunes, respectively; σ_x stands for the rms value of the horizontal electron-beam dimension.

It is evident that in the above equations σ_x plays a crucial role: an increase of the beam dimension provides a shift of the current threshold. In fact, such an increase may be induced by the FEL itself. We can indeed envisage the following mechanism: the FEL produces a beam heating, determining an increase of the energy spread, which is in turn responsible for an increase of the electron-beam transverse dimensions via a non-zero dispersion function at the octupole location.

As already remarked, this mechanism has been invoked in [1] to successfully account for the observed increase of Touscheck lifetime induced by the FEL onset and to explain the anomalous behaviour of the saturation dynamics in presence of a non-vanishing dispersion inside the undulator.

We can now give an idea of how the various elements can be combined together by simply reminding that, at equilibrium, the dimensionless FEL intracavity intensity and the square ratio of the induced to the natural energy spread are equal and determined by the equation [11]

$$\sqrt{1+X}\left[1 + 1.7\mu_\epsilon^2 (1+X)\right] = \frac{1}{r}, \tag{4}$$

with $X = (\sigma_i/\sigma_{\epsilon,n})^2$ and $r = \eta/(0.85 g_0)$. Here $\mu_\epsilon = 4\hat{N}\sigma_{\epsilon,n}$ ($\hat{N}$ being the number of undulator periods) stands for the inhomogeneous broadening parameter associated with the natural energy spread $\sigma_{\epsilon,n}$; σ_i is the FEL-induced energy spread, η represents the losses of the optical cavity and g_0 is the FEL small signal gain coefficient [12].

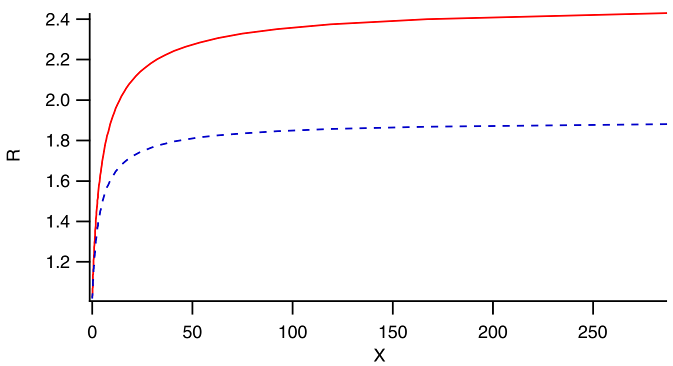

Fig. 2. $R = \left[\Im(\bar{Z})N_{th}\right]_{on} / \left[\Im(\bar{Z})N_{th}\right]_{off}$ versus the normalized FEL-induced energy spread $X = (\sigma_i/\sigma_{\epsilon,n})^2$. Here $\mu_\epsilon = 0.1$ and $A = 4$; $u = 2$ (continuous line) and $u = 1$ (dotted line).

The concurrent increase of the horizontal dimension is linked to X by the well-known relation [13]

$$\sigma_x = \sigma_x^0\sqrt{1 + A(1 + X)} \tag{5}$$

where A accounts for the dispersion function at the octupole location and σ_x^0 denotes the electron-beam horizontal dimension without the contribution of the dispersion function.

Combining equations (1)–(5), we can now make a comparison between the threshold current with and without the FEL. In Figure 2 we have reported the ratio $R = \left[\Im(\bar{Z})N_{th}\right]_{on} / \left[\Im(\bar{Z})N_{th}\right]_{off}$ as a function of the FEL-induced energy spread.

It can be seen that R is above unity when the FEL is on and increases for larger X values. This leads to a shift of the current threshold towards higher values. It might be argued that this conclusion is not correct, because we are not directly comparing the on and off threshold currents, but their products with the relevant imaginary parts of the effective impedance and we have no information on the behaviour of this last quantity. Even though the model does not provide such an information, we can observe, on physical grounds, that an increase of the beam dimension is always responsible for a weakening of the interaction between particles through the machine impedance. Therefore it can be always assumed

$$\Im(\bar{Z})_{on} < \Im(\bar{Z})_{off}, \tag{6}$$

thus enforcing the previous conclusion. In the following Section we generalize the obtained results.

3 FEL-induced stabilization and Keil-Schnell criterion

The statement that the FEL onset improves the effectiveness of the Landau damping can be generalized by invoking the so-called Keil-Schnell criterion [14]. According to it, the current threshold above which a collective instability is no more Landau damped and, therefore, is likely to manifest itself, is proportional to the beam (synchrotron or betatron) frequency spread $\Delta\nu$:

$$N_{th} \propto \Delta\nu. \tag{7}$$

The constant of proportionality depends on the electron-beam parameters and on the machine impedance. The link between the threshold current and the tune spread is a fairly general issue which can be extended to the quasi totality of electron-beam instabilities in storage rings (e.g. micro-wave instabilities [15]). Note that previous relation is valid, in the case of a bunched beam, both in the longitudinal and transverse planes [15]. In the preceding Section we have considered an example in which the FEL supports Landau damping in the cure of a transverse instability. We give now a more direct example to further corroborate the previous arguments. For this purpose we make use of the KS criterion to specify the threshold current of a generic (collective) longitudinal instability in the hypothesis of a coasting beam[1] having a given energy distribution and of a machine impedance characterized by a not strong reactive part [16,17]:

$$N_{th} = F\, T_0 \frac{2\pi E_0 \alpha_c}{(Z_n/n)e}\sigma_{\epsilon,n}^2. \tag{8}$$

Here Z_n/n is the broad-band impedance at the nth harmonic of the revolution frequency, α_c is the momentum compaction factor, E_0 the electron-beam energy, T_0 is the beam revolution period, and e is the electron charge. F is a form factor (close to unity) depending on the beam energy distribution. The extension of equation (8) to the case of a bunched beam can be performed by replacing T_0 with $\sqrt{2\pi}\sigma_t$, where σ_t is the rms value of the bunch temporal profile. Note that in this case N_{th} has to be interpreted as the bunch peak current. According to equation (8), when the average current N exceeds the threshold value, i.e. when $N > N_{th}$, the stability is not ensured. In this case the energy spread increases proportionally to $\sqrt{N}$ [18]. One therefore gets

$$\frac{N}{N_{th}} = \frac{\sigma^2}{\sigma_{\epsilon,n}^2} \equiv \delta^2. \tag{9}$$

As is well-known, the natural energy spread is a trade-off between the damping of the particles' motion in their longitudinal phase space (due to the emission of synchrotron radiation) and quantum diffusion effects (due to the stochastic character of such emission). An additional source of diffusion may contribute with an extra budget of energy spread, which combines quadratically with the natural one, thus shifting the threshold and eventually suppressing the instability. This is indeed the case of the FEL and according to equation (9) the FEL-induced energy spread necessary to counteract the instability is provided by

$$\sigma_i^2 = \left(\delta^2 - 1\right)\sigma_{\epsilon,n}^2, \tag{10}$$

[1] We perform the following analysis in the hypothesis of a coasting electron beam. In fact, while pulse propagation effects may influence the physics of single-pass FELs, their role can be generally neglected as far as SRFEL are concerned.

where we assumed $\sigma = \sqrt{\sigma_{\epsilon,n}^2 + \sigma_i^2}$. The current N defined by equation (9) becomes the new threshold. Now, in order to evaluate the ratio N/N_{th} for different initial conditions, one can start noting that the induced energy spread at equilibrium can be expressed as [18]

$$\sigma_i^2 = \frac{7.4 \times 10^{-2}}{\hat{N}^2} \frac{\tau_s}{T_0} \frac{I}{I_s}. \tag{11}$$

where

$$I_s \left[\frac{\mathrm{MW}}{\mathrm{cm}^2}\right] = 6.9 \times 10^2 \left(\frac{\gamma}{\hat{N}}\right)^4 \frac{1}{(\lambda_u[\mathrm{cm}] K f_b)^2} \tag{12}$$

and

$$f_b = J_0(\xi) - J_1(\xi), \qquad \xi = \frac{1}{4} \frac{K^2}{1 + K^2/2}. \tag{13}$$

Here I and I_s are the laser intracavity and saturation intensities (respectively), τ_s stands for the synchrotron damping time and K represents the undulator strength. By recalling that the saturation intensity, the FEL small-gain coefficient and the electron-beam power, P_E, are linked by the relation [11]

$$I_s g_0 \Sigma = \frac{P_E}{2\hat{N}} \tag{14}$$

(where Σ is the section of the optical mode) and that

$$P_E \frac{T_0}{\tau_s} = P_s \tag{15}$$

(where P_s is the power lost via synchrotron radiation by the beam during one machine turn), one gets, from equation (9), the following identity

$$\frac{N}{N_{th}} = \frac{I}{\bar{I}} + 1 \tag{16}$$

where

$$\bar{I} = \frac{P_s}{4\hat{N}} \frac{1.673\, \mu_\epsilon^2}{g_0 \Sigma}. \tag{17}$$

Equations (16) and (17) state that when the FEL is operating at an intracavity intensity I, the electron-beam current can exceed the KS threshold by the quantity $I/\bar{I}$. The equilibrium intracavity power may also expressed as [11]

$$I = \frac{P_s}{4\hat{N}} \frac{1.422\, \mu_\epsilon^2}{g_0 \Sigma} X. \tag{18}$$

Thus, finally,

$$\frac{N}{N_{th}} = 0.85X + 1. \tag{19}$$

Equation (19) may be exploited for estimating how much one can overcome N_{th} when the FEL is operating at equilibrium in a storage ring. A quantitative idea is given in Figure 3, where the ratio $m = N/N_{th}$ is reported as a function of the cavity losses. Note that m becomes quite large for small r values. Equation (19) has not been so far experimentally investigated in a systematic way. However, preliminary measurements carried out at Elettra have shown that when the FEL is on the instability threshold is increased. A detailed campaign of measurements, aimed at providing the scaling of N with the FEL intracavity power, is planned for the near future.

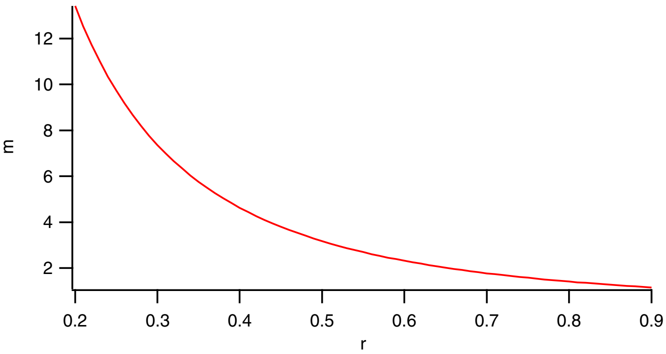

Fig. 3. $m = N/N_{th}$ versus the normalized cavity losses r for $\mu_\epsilon = 0.1$.

4 Conclusions

This paper can be considered as a point of arrival of a series of investigations aimed at clarifying the interplay between FEL, machine environment and non-linearities. The most significant result of these investigations has been that the FEL acts as a feedback mechanism for different type of instabilities and is an active element controlling the dynamics of the whole system. In this paper we have proved that the stabilization of the transverse and longitudinal beam dynamics can be, after all, traced back to the Landau damping reinforcement due to the FEL onset. It could be argued that SR dedicated to the production of synchrotron light normally work out of the operating regions of oscillator FELs, i.e. at relatively high energies (>1.5 GeV) and exploiting the emission of a large number of electron bunches. However, an increasing interest has been recently manifested, e.g. at ELETTRA [19], in the possibility of performing time-resolved experiments exploiting the light generated by few bunches at energies compatible with the FEL operation. In this case the use of a FEL oscillator could provide a flexible mechanism to control the stability of the whole machine. Finally, it is worth pointing out that even though our theoretical approach seems to be correct, as also proved by a number of experimental checks [1,6], the results obtained in this as well as in previous papers are influenced by the assumption that the FEL interaction does not significantly change the electron-beam energy distribution. This assumption can be considered correct only in first approximation. In fact, the FEL growth is not only responsible for a beam heating, with the consequent increase of the energy spread, but also for deformation of the distribution [20]. Even though this effect does not modify the main conclusions of our analysis, deviations from equation (19) may occur as it will be carefully discussed in a forthcoming paper.

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